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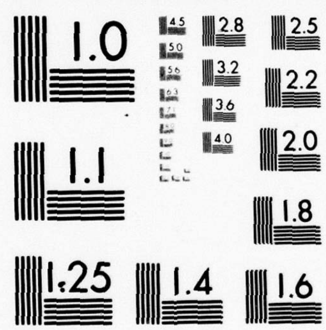

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FOUR RUSSIAN ARTICLES ON OCEANIC TURBULENCE, CURRENT VARIABILITY, AND PERIODIC INTERNAL WAVES IN THE BLACK SEA

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12 August 1964

Translated by
C. N. K. Mooers

U. S. NAVY UNDERWATER SOUND LABORATORY
FORT TRUMBULL, NEW LONDON, CONNECTICUT

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FOUR RUSSIAN ARTICLES ON OCEANIC TURBULENCE, CURRENT VARIABILITY, AND PERIODIC INTERNAL WAVES IN THE BLACK SEA

~~Translated by~~

10 C. N. K. Mooers

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PREFACE

Several recent Russian oceanographic contributions have been translated; they are germane to the topics of ocean dynamics, e.g., turbulence and internal waves. The article by R. V. Ozmidov on oceanic turbulence is one of the clearest discussions of the topic to date, and it also presents new quantitative results. The application of the theory of random function to ocean currents by B. N. Belyaev and U. S. Boldirev suggests avenues of approach to the prediction of ocean currents where a good statistical knowledge of the current field is available. The other articles give further theoretical and experimental considerations related to periodic motion in the sea.

The intrinsic value of this material is twofold: it presents techniques and data. Its significance to underwater acoustic systems and research is obviously that some of the variability observed in acoustic transmission data may be attributed to motions of the medium; if this variability is to be understood, some knowledge of the medium variability must be achieved.

16 SR00403

The four articles contained in this memorandum are entitled:

Some Aspects of Oceanic Turbulence (p. 3)

17 SR0040307

Application of the Theory of Random Functions to the Study of Ocean Currents (p. 27)

254 200

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Some Results of Investigations of Tidal Currents in the Open Parts of the Pacific Ocean (p. 43)

On the Periods of Internal Waves in a Deep Inland Sea (p. 46)

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SOME ASPECTS OF OCEANIC TURBULENCE*

by

R. V. Ozmidov

Institute of Oceanology, Academy of Sciences, USSR

ABSTRACT

Semi-empirical and statistical methods for the investigation of oceanic turbulence are discussed. The first method concerns the difficult problems of averaging different characteristic fields in the sea and of determining virtual exchange coefficients. These exchange coefficients depend critically upon the scale of phenomena involved. The second method has appeared in oceanographic practice only in recent years. Results are given for the calculation and analysis of the statistical characteristics of large-scale turbulence in the Black Sea. Data of current speeds from multi-day stations at the 20 and 100 meter horizons are used. The following computations are made: turbulent intensity, tensor components of correlation moment, correlation coefficient, autocorrelation coefficient, and spectral and structural functions. This analysis leads to conclusions about the anisotropy of oceanic turbulence and the ratio of inertial to turbulent motions in the sea. The amplitude of inertial oscillations and the rate of dissipation of turbulent energy are estimated.

If any sensitive device for the measurement of current speeds is lowered to some depth in a region of the sea, described on a chart as a region of constant current (directed for example to the north), and if the device is held at the sample point for a few minutes, then,

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after reading it, there will be a high probability that the device will show that the currents are by no means directed to the north and that the absolute magnitude is substantially different from that anticipated. A measurement of speeds at that same point in the sea at some other moment of time may give a different result from the former and may again not coincide with that inferred from the data on the chart. Only by producing a lengthy series of observations and averaging them may we hope to acquire a chart which is close to what is displayed on our chart. That example shows that constant (or slowly and regularly changing) currents in reality do not exist in the seas or oceans at a given moment of time, because they are the result of the application of an averaging operation to actual instanteous currents in natural fields of turbulent velocities. Reynolds (O. Reynolds, 1895) suggested that the representation of components of such instantaneous turbulent speeds be in the form of the following sums:

$$u=\bar{u}+u'\,;\; v=\bar{v}+v'\,;\; w=\bar{w}+w' \qquad (1)$$

where $\bar{u}$, $\bar{v}$ and $\bar{w}$ are mean values, and u', v' and w' are the fluctuating (turbulent) components of speed.

By "mean speed", an expression of the following form is ordinarily understood:

$$\bar{u}=\frac{1}{T}\int_0^T u(t)\,dt \qquad (2)$$

where T is the averaging period. Similar to such time averages, an integration can be performed on any spatial field averaged over G, a linear dimension.

Obviously, as T increases, more large-scale pulsations of speeds are included in the averaging. If this averaging period significantly surpasses the duration of all the flow oscillations, then the average value will not be a function of time, and repeated averages will not change their values. Consequently, in that case, the following relationships will be true:

$$\bar{\bar{u}}=\bar{u}\,;\; \bar{\bar{v}}=\bar{v}\,;\; \bar{\bar{w}}=\bar{w}\,;\; \bar{u}'=\bar{v}'=\bar{w}'=0 \qquad (3)$$

Such a theory for averaging fields of speeds is related to the usual theory of the distribution and transmission of energy on various scales of motion in the ocean. It is believed that energy from surface sources (tangential wind friction, or nonuniform heating of water masses by the sun) causes the large-scale mean movement, which in its turn supplies energy to smaller structures in the flow, and so on, to the smallest turbulent vortices, in which there occurs a dissipation of the mechanical energy into thermal motion in the liquid. That would seem to be the natural scheme, serving as the basis for many theoretical constructions; however, we do not have sufficient experimental confirmation concerning conditions in the ocean, and this theory has even been doubted in recent years. Stommel (H. Stommel, 1949) considered that the influx of energy to oceanic currents from the tangential friction of wind existed not only for mean flow but for the entire spectrum of turbulent oscillations. Moreover, recently a supposition has been expressed about the possibility of a reverse flow of energy, i.e., from large turbulent eddies in the ocean (meanders) to the mean current. Confirmation of such a possibility was recently introduced by Webster (F. Webster, 1961), who calculated the transport of kinetic energy from large vortices (meanders) to the mean current of the Gulf Stream. This placed in doubt what is ordinarily accepted in the theory of ocean currents as an axiomatic statement concerning the analysis of external forces acting on a given water mass, i.e., that we ought to obtain from the average equations of motion for fluids some regular (either stationary or regular depending upon the relevant forces) speed field in the observed region of the ocean. Realistically, if there is initially a chaotic transfer of turbulent oscillations (even though they would be of a large scale), one must apply what is termed the "average" operation to the current and test for the irregular variations, which may not be used for evaluating the average equations of motion. In that case, numerous theoretical statements concerning oceanic currents and turbulent exchange, which are used for the average equations of motion, diffusion, and heat conductivity ought to be adaptable to the theoretical determination of the actual fields of analyzed values through the instantaneous functions. But, for the condition of validity for ordinary schemes of turbulent oceanic motions, i.e., that there exists an energy balance, the knowledge of either the speed or other average characteristic fields (temperature, salinity, pressure, etc.) is often found quite insufficient for solving many problems. So for the prediction of the trajectory of motion of a ship in an associated current body (e.g., for a drifting ship), it is necessary, obviously, to have information about the characteristics of the instantaneous, specific fields which move this body.

Investigations into the characteristics of instantaneous turbulent speeds in the ocean began to develop in recent years, and the direct results achieved are still quite modest. Moreover, expressions describing the field of turbulent speed are very complicated, and difficulties of a fundamental character arise in their solution. For this reason, the equations are ordinarily averaged in some mode and brought into concurrence with the form of the equations for laminar mean flow, and the effect of turbulence is accounted for by the introduction of some virtual coefficients of turbulence. These coefficients, which may be determined from the equations of motion, do not appear to be physical constants and depend upon the characteristics of the flow and, most important, upon the period (or scale) of the average used in each problem. That consideration does not have special significance for those turbulent flows in which there are maximum periods of turbulent micro-pulsations, i.e., whose averages become stationary without further increase of the averaging period. Oceanic turbulence is a considerably complex case. Here it is difficult to identify the value of T over which all the turbulent pulsations in the sea would be eliminated. It follows that, in the sea, the choice of averaging period T (or scale for a spatial average) plays a determining role in the empirically derived coefficients of turbulence. However, in the initial studies of turbulence in the sea, when coefficients of turbulence were not determined in the basic measurements of fluctuations and investigations of magnitudes, but by the method of observing averaged fields of these quantities, questions about scales of averages (scales of phenomena) did not occupy a significant influence. Meanwhile, just the differences in scales of averages (phenomena) were, to a significant degree, the explanation for the enormous diversity in the magnitudes of the coefficients of turbulent exchange (vertical as well as horizontal) acquired by various authors for the different regions of the world ocean.

Some consideration is required regarding the above described method of constructing and solving the problem of the dynamics of turbulent motions and the processes of turbulent exchange, by which the semi-empirical coefficients of turbulence are determined. It is to be noted that most of the major problems in the dynamics of sea currents and turbulent exchange in the sea are at present determined practically only in the limits of such semi-empirical theories. That circumstance explains the limitations of most of the works on oceanic turbulent exchange (momentum, heat, salt, etc.), which are devoted to various indirect methods of determining turbulent coefficients. It is hardly expedient to enumerate these works because the majority of them suffer by being largely insufficient due to the absence of data about the relationship between the coefficients of turbulence found from the

determined parameters of each specific problem and the scale of average used in their solution. We come only briefly to a series of works in which this insufficiency is absent and which is devoted to investigations into the dependence of exchange coefficients upon scale of the average (of the phenomenon).

These investigations were principally made through observations of the motion of discrete particles in associated oceanic currents. If data about the positions of a series of such particles at successive moments of time exist, the coefficient of turbulent exchange (diffusion) may be determined by the formula:

$$A(l)=\frac{\overline{(\Delta l)^2}}{2\Delta t} \qquad (4)$$

where Δl is the change in distance l (scale of phenomenon) between individual pairs of particles occurring in the time Δt. The bar indicates an average determined over a series of individual observations.

The first of these experiments in the sea were achieved by Richardson and Stommel (L. H. Richardson, H. Stommel, 1948; H. Stommel, 1949) and repeated on a broader interval of scale change (up to $l = 10^5$ cm) by R. V. Ozmidov (1957, 1959). These experiments showed that the coefficient of turbulence in the sea really depends upon the scale of the phenomenon, for that dependence is expressed as the ideal 4/3 power law, i.e.,

$$A(l)=\varkappa l^{4/3} \qquad (5)$$

where $\varkappa$ is the coefficient of proportionality, which is approximately 0.01 $cm^{2/3}$ sec^{-1}. Similar values of $\varkappa$ for average conditions existing in the sea were found by Inoue (E. Inoue, 1952a), Defant (A. Defant, 1953), and Hanzowa (M. Hanzowa, 1953.

Equation (5), obtained experimentally for the coefficient of turbulent exchange, did not appear unexpectedly, because as early as 1941 A. M. Obukov acquired, for the coefficient of turbulent exchange from dimensional considerations, the "4/3 power" law, which is based on a theory for the onset of turbulent flow proposed by A. N. Kolmogorov.

In 1960 Ichye and Olson (T. Ichye, F. C. W. Olson, 1960) made an attempt to extend the "4/3 power" law to processes with a scale of up to 1000 km. At the same time, Joseph and Sendner (J. Joseph, H. Sendner, 1958) and similarly Schönfeld (J. Schönfeld, 1961) hypothesized that for such macroscopic processes in the ocean the "4/3 power" law loses strength, and in that case, the coefficient of exchange is best related to l, i.e., a linear dependence exists. That question, obviously, still requires further investigation; however, even now, with the dependence of $A(l)$ on the scale of phenomenon established, there is the possibility of being more correct than formerly in the selection of values for the exchange coefficient for the solution of various problems about oceanic currents and processes of exchange.

Other recently developed investigations of turbulence in the sea are based on the statistical and spectral theory of turbulence, and their principal attention is devoted not to investigations of average fields of various substances and speeds in the ocean, but to the study of instantaneous, fluctuating values of these quantities. This approach is based upon the representation of turbulent flows as the superposition of many scales of fluctuations, i.e., the generators of the distinct turbulent spectrum. In the semi-empirical theories, turbulence is not actually investigated; thus the nature of turbulent exchange coefficients remains vague to a significant degree.

The distinct statistical characteristics of turbulence have been acquired to a sufficiently broad extent in recent years by investigations of turbulent flows in aerodynamic tunnels and other laboratory equipment, and also in layers of the earth's atmosphere. Unfortunately, in investigations of oceanic turbulence, the statistical methods of investigation are quite poorly developed. But, before dwelling on the works bearing on that question, the requirements for instrumentation for measuring fluctuations of quantities in a field of oceanic turbulence will be briefly described.

If the average speeds of currents have small values and if the periods of fluctuations are likewise not great, for measuring the fluctuations of speeds, it is obviously necessary to have a quite precise and low-inertia apparatus. This is particularly germane to the study of vertical turbulence, which is a case of extremely small average speed, and wherein the scale of the phenomenon is also comparatively small.

The case is somewhat different for investigations of horizontal turbulence in the sea. The huge horizontal dimensions of the seas and oceans provide extremely large-scale horizontal processes. As for the average horizontal speeds, so the fluctuations of these speeds have incomparably larger magnitudes than the analogous vertical characteristics. For the study of horizontal turbulence in the sea, the requirements for precision and "inertialness" of the apparatus employed ought to differ from those used for measuring the characteristics of vertical turbulence. If in the latter case a sufficient representation of average values and fluctuations of speeds is acquired by records from low-inertia devices over a period of a few minutes, then for studies of horizontal turbulence, in order to be able to more or less envelop large-scale changes in horizontal speed, it is necessary to have lengthy series of observations (on the order of several days). But the inertial and precision requirements of these instruments may be significantly relaxed. So devices which record speeds averaged for 2 - 3 minutes as "instantaneous speeds" are recognized as fully satisfactory for the investigation of horizontal turbulence in the sea, if, of course, one is not interested in the especially subtle microstructure of fluctuations, which give only insignificant contributions to the general energy of horizontal turbulence in the sea. For studies of the vertical turbulence, such devices are useless and a low-inertia device of one type or another must be designed.

A well-known success in this direction was attained in the Department of Marine Physics of the Moscow Government University under the direction of A. G. Kolesnikov (1958, 1959c). With the aid of the low-inertia devices (turbulence meters), measurements of fluctuations of speeds and temperatures were conducted in the cold-water layer in Lake Baikal and at the stations of "North Pole-4". This work was also conducted in the Indian and Atlantic Oceans and the Black Sea. As a result of the analysis achieved, the papers of A. G. Kolesnikov and his co-workers(1959a, b, 1960; N. A. Panteleev, 1959) acquired curves for the distribution according to depth (z) of turbulent intensity, coefficients of turbulent viscosity $\nu(z)$, and thermal conductivity $k\ (z)$. It was shown that, in conditions of stratification, the average magnitude of $\nu(z)$ is many times larger than the value of $k\ (z)$, so that on the average $\nu(z) = 50 \cdot k\ (z)$. They also discovered the dependence of the coefficients $\nu(z)$ and $k\ (z)$ upon the stability criterion for layers of water; however, a precise, analytical form for this dependence was not achieved. A. G. Kolesnikov also succeeded in confirming, in sea conditions, the well-known law of A. N. Kolmogorov concerning the change in speeds of turbulent fluid motion with distance

according to a power law, with a demonstrated power of 1/3. It is to be noted that all these results were acquired in the surface layers of the sea, while the continuous determination of corresponding quantities did not exceed several minutes.

A device for measuring speed fluctuations, based on the electromagnetic method, was applied by Bowden and Fairbairn (K. F. Bowden, L. A. Fairbairn, 1952) to the investigation of turbulent fluctuations of the horizontal speed u in strong tidal currents at two meters above the bottom. Calculations of autocorrelation and spectral functions for the u component of speed were produced, and estimates of the scale of turbulence were also made. In other work, Bowden and Fairbairn (K. F. Bowden, L. A. Fairbairn, 1956) described experiments of simultaneous measurements of speed fluctuations u and w, that made it possible for them to compute the Reynolds' stress: $\rho\overline{u'w'}$. However, the length of the records of speeds in the experiments of Bowden and Fairbairn did not exceed 5 - 10 minutes because they acquired values related to the microstructure rather than the large-scale structure; but this flow, as already shown, has a highly specialized form.

Turbulently pulsating speeds in current flows near Cape Cannon were measured by Nan'niti (T. Nan'niti, 1956) with the help of his invention of a photoelectric instrument for measuring currents. It was revealed that the ratio of average amplitudes of pulsation to the magnitude of average speed (the intensity of turbulence) was maximum near the bottom and minimum in the middle of the flow. Computation of the autocorrelation function R(t) showed that it maintains an absolute law for various series of observations and that periods averaged from 10 - 20 seconds to 5 - 10 minutes. The theoretical expression for the autocorrelation: $1 - R(t) \sim t^{2/3}$, follows from similarity theory, which is rarely fulfilled by data of the experiments described. For this reason, in the opinion of Nan'niti, it appeared that the duration of the observations and period of averaging were insufficient, and, likewise, that the sensitivity of the device was low.

A low-inertia device of the thermal anemometer type for the measurement of turbulently pulsating speeds was applied by Grant, Stewart, and Moillet (H. L. Grant, R. W. Stewart, A. Moillet, 1960) in a strong tidal current in Discovery Straits on the coast of British Columbia. The device measured turbulent fluctuations with frequencies up to one kilocycle, so that for average speeds of flow of 100 cm/sec, there

corresponded a minimum dimension of 1 millimeter for the decay of turbulent motion. The spectral function for the investigated turbulent flows was computed, and it was shown that these spectra correspond to the theoretical expressions of A. N. Kolmogorov and Heisenberg.

In addition to the enumerated investigations of the microstructure of the speed field in oceanic turbulence, there is a series of works dedicated to the study of pulsating water temperatures (L. Liberman, 1951; E. Inoue, 1952; W. W. Englich, 1953; T. Nan'niti, 1957; N. V. Kontoboyzeva, 1958; and others). These works revealed a series of laws of temperature pulsations in the sea (especially in the form of the autocorrelation function); these laws are important for the specific investigation of turbulent heat conduction and the formation of temperature fields in the sea.

A significantly smaller effort has been dedicated to the investigation of horizontal macroturbulence in the sea through the aid of observations over horizontal speeds of currents by ordinary oceanographic rotor devices. The first works in this province, as also, in general, in all regions of the application of statistical methods to the study of oceanic turbulence, appeared in the investigations of V. B. Shtokman, accomplished in the years 1940 - 1941. In the first work (1940), V. B. Shtokman determined coefficients of horizontal turbulent friction in the Caspian Sea using data of current speeds measured by a hydrological rotor; 10-minute averages were formed every two hours in the course of 23 hours. Coefficients of horizontal turbulent friction were computed by the formulae of Ertel, which were generalized expressions of the semi-empirical theory of Prandtl in tensor form. By a long path of synthesis, V. B. Shtokman determined the path of integration for pulsating speeds in the limit of two successive changes of its law. The value of the coefficient of horizontal turbulent exchange appeared to lie within the bounds of 1.2 to 4.9 X 10^6 cm^2/sec, with an abrupt change of value for anisotropic exchange relative to the direction of the coast line.

In his work of 1941, V. B. Shtokman used the following formula for the determination of the horizontal exchange coefficient, which follows from the averaged equations of fluid motion:

$$A_{ik} = \frac{\overline{\rho u_j' u_k'}}{\frac{\partial \bar{u}_i}{\partial x_j}} \qquad (6)$$

With this formula, V. B. Shtokman found A_{xx} = 55 X 10^6 cm^2/sec and A_{yy} = 7 X 10^6 cm^2/sec (the x axis was directed along the coast line).

The simultaneous measurement of current speeds with several rotors at a series of points in the vertical enabled V. B. Shotkman to compute the coefficient of $R_u(z)$ (z = the vertical coordinate). As a result, he obtained the characteristic curve of correlation, which affirms that this law is approximately preserved to a depth of 25 meters (the sea depth was 35 meters at this place).

Formula (6) is the same as that used by Stommel (H. Stommel, 1955) for the computation of the coefficient of horizontal turbulent exchange in the Florida Straits by data of speeds measured at 15-minute intervals from an anchored vessel in 1885. Analogous computations were produced for the region of the Kuroshio Current (T. Ichye, 1957), using a survey of speeds produced by the aid of an EMIT (GEK).

Works, repeating to a significant degree the statements of the earlier investigations of V. B. Shtokman, were acquired for the Black Sea by A. N. Gezentzvey (1961) and V. V. Hlopov. Moreover, A. N. Gezentzvey (1959) examined the question about the period of averaging fluctuating speeds on the acquired values of exchange coefficients.

That short enumeration of work is limited, unfortunately, to a list of investigations devoted to the study of the characteristics of horizontal turbulence in the sea on the basis of the measurement of pulsating speeds. Determination of these statistical characteristics of horizontal macroturbulence in the sea, with the exception of the coefficients of exchange and some magnitudes computed by V. B. Shtokman, are virtually unrealized. For that reason, such work was undertaken by the author in the Dynamics of the Sea Laboratory, Institute of Oceanology, AN CCCP* (R. V. Ozmidov, 1962).

In the capacity of initial material for the determination of statistical characteristics of horizontal macroturbulence, we used data which was measured by current speed rotors for many days at buoy stations and taken in the Black Sea in 1956. Buoy stations with

*Translator's Note: Academy of Sciences, USSR.

two rotors of the BPV-2 system of Alexseyev at 20 and 100 meter horizons were deployed at a point with coordinates 42° 50' N and 40° 25' E and maintained from 13 to 27 August. The interval between the sampling of rotor speeds was established at 20 minutes. Altogether, the tapes of speed records consist of 1008 imprints of vector values of horizontal current speeds for the 20-meter horizon and 1009 imprints for the 100-meter horizon of observations.

By analysis of the acquired tapes of current speeds, it was not difficult to display the investigated speed field in the presence of irregular oscillations of very differing magnitudes, beginning from small scales, which provoked the analysis of successive values of speed, and finally the very largest scale with periods commensurable with the length of the entire interval of observation. These large-scale changes of speed may, of course, be viewed as "regularities" and may be studied for the analysis of reasons for the origin of the flow and its variability (for example, under the action of a changing wind field). However, there is the possibility of other points of view, in accordance with which changes of vector speeds of all scales may be observed as "random" pulsations, subject only to investigation by statistical methods. The first such view of large-scale processes in the atmosphere was apparently developed by Defant (A. Defant, 1921); he suggested viewing even such large formations in the atmosphere as cyclones as elements of atmospheric macroturbulence. A similar point of view was first, and also quite fruitfully, developed for horizontal turbulence in the sea by V. B. Shtokman in the previously cited work. For the computation of statistical characteristics of horizontal turbulence in the Black Sea, we proceed to those representations which view all speed oscillations for all dates of observation as random turbulent pulsations of various scales.

In that case, the speed component of currents in the investigated flow is naturally represented in accordance with formula (1) in the form of a sum of averaged and pulsating speeds.

Computation of the average speeds $\bar{u}$ and $\bar{v}$ for all periods of observation (14 days) gave the following magnitudes (with an accuracy of 1 cm/sec): $\bar{u} = 0$, $\bar{v} = 8$ cm/sec for the 20-meter horizon; and $\bar{u} = 0$, $\bar{v} = 7$ cm/sec for the 100-meter horizon of observation. Since the average value of fluctuating currents $\overline{u'}$ and $\overline{v'}$ are by their definition equal to zero, as a measure of these amplitudes, there were computed mean square values of the fluctuating deflections.

Computation of these magnitudes from all the fluctuating speed values possessed for the 20-meter horizon gave the following figures:

$\sqrt{\overline{(u')^2}}$ = 16.9 cm/sec and $\sqrt{\overline{(v')^2}}$ = 12.8 cm/sec.
Corresponding magnitudes for the 100-meter horizons of observation showed the following: $\sqrt{\overline{(u')^2}}$ = 12.6 cm/sec and $\sqrt{\overline{(v')^2}}$ = 7.6 cm/sec. Dividing the magnitudes of the mean square value of speed fluctuations by the average speed of flow, we acquire an important statistical characteristic of turbulence, i.e., its intensity (longitudinal and transverse): I_u = 2.1, I_v = 1.6 for the 20-meter horizon and I_u=1.8, I_v = 1.1 for the 100-meter horizon. These figures show that the intensity of turbulence in the period of observation was quite high, and this turbulence was not completely isotropic, since its longitudinal intensity was less than its transverse. As anticipated the intensity of turbulence decays with depth -- at the 100-meter horizon it is significantly below the intensity on the upper or 20-meter horizon.

The four different combinations of component products of fluctuating speeds form a symmetrical tensor of correlation moments, the significant components of which, $\overline{u'^2}$ and $\overline{v'^2}$, were adduced formerly; then the components $\overline{u'v'}=\overline{v'u'}$ were computed and gave the values 89 cm/sec and 68 cm/sec* for the 20- and 100-meter horizons, respectively. The fact that the non-diagonal members of the tensor of the correlation moments turned out to be different from zero again confirms the anisotropy of the turbulent flow.

As a measure of the statistical tie between data of pulsating magnitudes (u' and v' in our case), the so-called correlation coefficient R is ordinarily used; it is formed from the corresponding correlation moment divided by the mean square deviation of the observed pulsating magnitudes. Computation of correlation coefficients gave for the 20-meter horizon R =-0.41 and for 100-meter horizon R = -0.72. It follows that the pulsating components of speed were more random (chaotic) on the upper horizon than on the lower, where they may have a closer relation with one another.

*Translator's Note: It is believed that the units intended were $(cm/sec)^2$ vice cm/sec.

In addition to the previously mentioned correlation moments, correlation moments were computed between one and another of these components of pulsating speeds, measured at one point, but at a different moment of time:

$$B_u(t) = \overline{u'(t_0) \cdot u'(t_0+t)}\,;$$
$$B_v(t) = \overline{v'(t_0) \cdot v'(t_0+t)} \qquad (7)$$

where t_0 is some moment of time taken as the initial time, and may have any value.

Since the initial material presented was not a continuous record of speed but a series of discrete values, the computed correlation moments (7) were not produced by integration in time but by means of a summation over a series of individual samples of the flow, sampling from some beginning reading through a series of successive moments of time on the tape record of speeds. The number of the many-valued functions $B_u(t)$ or $B_v(t)$ necessary for an average must be foreseen in advance to avoid difficulties, and for us it was selected as equal to 100, i.e., each value of correlation moment was acquired as a result of averaging over 100 individual values of the product with the form $u'(t_0) \cdot u'(t_0+t)$. For the investigation of the question about the significance of the number of individual values of the function $B_u(t)$, used in the average, we produced a special computation of parts of the function $B_u(t)$ for different multiples of the sampling period. That computation showed that, when the total ~~lag~~ period (t) equalled from 50 - 75 times the averaging period, the function $B_u(t)$ was fully developed and did not change by a further increase of the lag interval.

For the clearest graphs of the computed correlation moment functions, all values of these functions were normalized to unity, i.e., divided by the product of the mean square magnitudes of corresponding fluctuating speeds in initial and in sampled moments of time:

$$R_u(t)=\frac{\overline{u'(t_0)\cdot u'(t_0+t)}}{\sqrt{\overline{[u'(t_0)]^2}}\ \sqrt{\overline{[u'(t_0+t)]^2}}};$$
$$R_v(t)=\frac{\overline{v'(t_0)\cdot v'(t_0+t)}}{\sqrt{\overline{[v'(t_0)]^2}}\ \sqrt{\overline{[v'(t_0+t)]^2}}} \qquad (8)$$

The functions $R_u(t)$ and $R_v(t)$ were computed over 100 samples for all periods of observation and for the 20- and 100-meter horizons. Graphs thus acquired show that pulsating speeds are correlated with themselves (autocorrelation) greatly over lengthy intervals of time. Indeed, the curves for $R_u(t)$ and $R_v(t)$ for the 20-meter horizon approach zero as a limit for the first time at t = 135 hours and t = 81.5 hours, respectively. For the 100-meter horizon, the positive law of functions $R_u(t)$ and $R_v(t)$ is preserved even further: 141.5 and 150 hours. But with the expiration of such a large interval of time the absolute values of the functions $R_u(t)$ and $R_v(t)$ do not become small but become large negative values, and only at the end of the observation interval when the argument $t \sim$ 12 days do the functions $R_u(t)$ and $R_v(t)$ again approach zero. It is of interest to note that, for the 20-meter horizon, the first point after the initial point of functions $R_u(t)$ and R $R_v(t)$ shows a fall in correlation of 26 percent for the u component of speed and of 17 percent for the v component. Likewise, for the 100-meter horizon the drop consists of 7 and 2 percent, respectively. That fact states that, in the upper layer of the sea, a comparatively large role is played by small period ($t <$ 20 minute) fluctuations, but fluctuations for $t >$ 20 also influence the autocorrelation relationships of the given speed components. At the greatest of these depths, such short period speed fluctuations are developed quite weakly.

In addition, for the basic oscillations with a period close to the periods observed on all graphs of functions $R_u(t)$ and $R_v(t)$ for the 20-meter horizon, it is not difficult to display (especially in the middle part of the graphs) oscillations with periods near 17 hours, i.e., near to the period of inertial oscillations which may exist in the given region of the sea. For a clearer elucidation of these oscillations, we computed the functions $R_u(t)$ and $R_v(t)$

for the 20-meter horizon by choosing the starting moments of time very close to the beginning of the rise of the phenomenon studied. For that purpose, as initial moments of time, marks on the tape recording numbered 389 - 488 were taken. In this manner, inertial oscillations became obvious and the autocorrelation functions yield a highly accurate view of the oscillating process with decaying amplitude (Fig. 1). The period of this oscillation equals 16 hours and 40 minutes, i.e., it corresponds exactly to the inertial period for the latitude of the observation points. Consequently, the construction of such statistical characteristics of flows as autocorrelation functions makes it possible to discover the presence of orderly movements of one or another period in turbulent flows. Some theoretical aspects of the question were examined by describing other statistical characteristics of turbulence, namely, the so-called structure functions which also make it possible to discover the presence of periodic oscillations with inertial period.

As already shown, the turbulent flow studied did not appear isotropic, as might have been expected; thus, clearly, the large-scale vortex formations in the sea ought, by one means or another, to depend on the geometrical characteristics of the flow (coastline features, bottom relief). In the capacity of a statistical characteristic of such anisotropic flows one ordinarily uses, as first introduced by A. N. Kolmogorov, the structure functions, determined by the following expression:

$$D_{jk}=\overline{[v_j(t_0+t)-v_j(t_0)][v_k(t_0+t)-v_k(t_0)]} \qquad (9)$$

where j , k equal 1, 2, or 3, and v_j and v_k are any of the speed components.

If now it is assumed that the interval of time t does not exceed scale τ , less than which the vortices may be considered isotropic, then the only two non-zero structure functions are shown to be:

$$D_u(t)=\overline{[u(t_0+t)-u(t_0)]^2};$$
$$D_v(t)=\overline{[v(t_0+t)-v(t_0)]^2} \qquad (10)$$

In scale intervals less than r and greater than r_0, where the action is determined by viscous forces, for structure functions $D_u(t)$ and $D_v(t)$, it may be found from dimensional considerations that:

$$D_u(t)=c_1\varepsilon^{2/3}U^{2/3}t^{2/3};\ D_v(t)=c_2\varepsilon^{2/3}U^{2/3}t^{2/3} \qquad (11)$$

where c_1 and c_2 are some universal dimensionless constants on the order of unity, ε is the rate of dissipation of turbulent energy in the flow, and the product Ut has the sense of "distance" between the "points" at which the measurements are produced.

Computations of the structure functions $D_u(t)$ and $D_v(t)$ were produced for both components of speed and for the 20-meter as well as the 100-meter horizon with the use of 100 samples in averaging. The results showed that the structure functions of the speed fields in the sea satisfactorily agreed with formulae (11). Moreover, there are superimposed on the general dependence of the "2/3 law" some large-scale oscillations, e.g., the periodic oscillations with an inertial period of 16 hours and 40 minutes, as occurred for the autocorrelation function. We will examine that question in more detail. We assume that in the sea, side by side with an isotropic field of chaotic turbulent speeds $v(t)$, there exist regular components of speed, changing according to a periodic law: $A\cos\omega t$; that is, the total speed of flow would be:

$$V(t)=A\cos\omega t+v(t) \qquad (12)$$

Constructing the structure function for such a speed field, we have:

$$D_v(t) = \overline{[V(t_0+t) - V(t_0)]^2}$$
$$= \overline{[A\cos\omega(t_0+t) - A\cos\omega t_0]^2}$$
$$+ \overline{[v(t_0+t) - v(t_0)]^2} \qquad (13)$$

(The term $2\overline{[A\cos\omega(t_0+t) - A\cos\omega t_0]\cdot[v(t_0+t) - \overline{v}(t_0)]}$ equals zero as a resultant property of averaging).

Averaging again over a period T, i.e, integrating t * from 0 to T, we will have:

$$D_v(t) = \frac{A^2}{T}\left\{\left[\frac{T}{2} + \frac{1}{4\omega}(\sin 2\omega(T+t) - \sin 2\omega t)\right]\right.$$
$$-\left[2\cos\omega t\left(\frac{T}{2} + \frac{1}{4\omega}\sin 2\omega T\right)\right.$$
$$\left.-\left(2\sin\omega t\cdot\frac{1}{2\omega}\sin^2\omega T\right)\right] \qquad (14)$$
$$\left.+\left[\frac{T}{2} + \frac{1}{4\omega}\sin 2\omega T\right]\right\} + d_v(t)$$

where $d_v(t)$ is the structure function of the field of random turbulent $v(t)$.

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*Translator's Note: The original paper gave t where it should be t_0.

If the averaging period T is a multiple of the period of oscillation of the regular speed component (which occurs in our case), then we acquire:

$$D_v(t) = 2A^2 \sin^2 \frac{\omega t}{2} + d_v(t) \tag{15}$$

The structure function $d_v(t)$ for the turbulent speed field $v(t)$ ought to obey the "2/3 law". Consequently, we have the conclusion:

$$D_v(t) = 2A^2 \sin^2 \frac{\omega T}{2} + c_2 \varepsilon^{2/3} U^{2/3} t^{2/3} \tag{16*}$$

The graph of the structure function computed by this method is presented in Fig. 2 in which experimental points of the naturally occurring structure function for the v component of speed at the 100-meter horizon are also plotted. The graph of function $D_v(t)$ is constructed by formula (16) for the choice of the following values as its constants: $A = 2.7$ cm/sec, $c_2 \varepsilon^{2/3} U^{2/3} = 1.72 \times 10^{-2}$ cm^2/sec$^{8/3}$. The magnitude of A is determined by the amplitude of inertial oscillations existing at the time of observations in the region of investigation in the Black Sea. The acquired value for the magnitude of A is in good agreement with the value of the amplitude of inertial oscillation calculated by A. D. Yampolski (1960) with the data of these observations, but by using a method different from that just described. The values of the two constants in formula (16) make it possible to produce an estimate of an important characteristic of turbulence: the rate of dissipation of turbulent energy ε.

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*Translator's Note: The $2A^2 \sin^2 \frac{\omega T}{2}$ term appeared in the original paper but should be $2A^2 \sin^2 \omega t/2$.

Considering that the mean speed U for the investigated segments of time was equal to 17.0 cm/sec, and further assuming that $c = 1$, we find the value of 1.32×10^{-4} cm^2/sec^3 for ε. Although direct measurements of ε at a depth of 100 meters were not produced at this time, the order of magnitude of ε acquired by us agrees with the available indirectly estimated value of ε in the sea. (R. V. Ozmidov, 1960).

The good agreement between the theoretical curve and experimental data for the structure function and, likewise, the realistic acquired values for the magnitudes of A and ε once again show the fruitfulness of the application of statistical methods of the theory of turbulence for investigations of the various dynamic processes existing in the sea. And, undoubtedly, these methods will find a quite broad application in oceanographic practice.

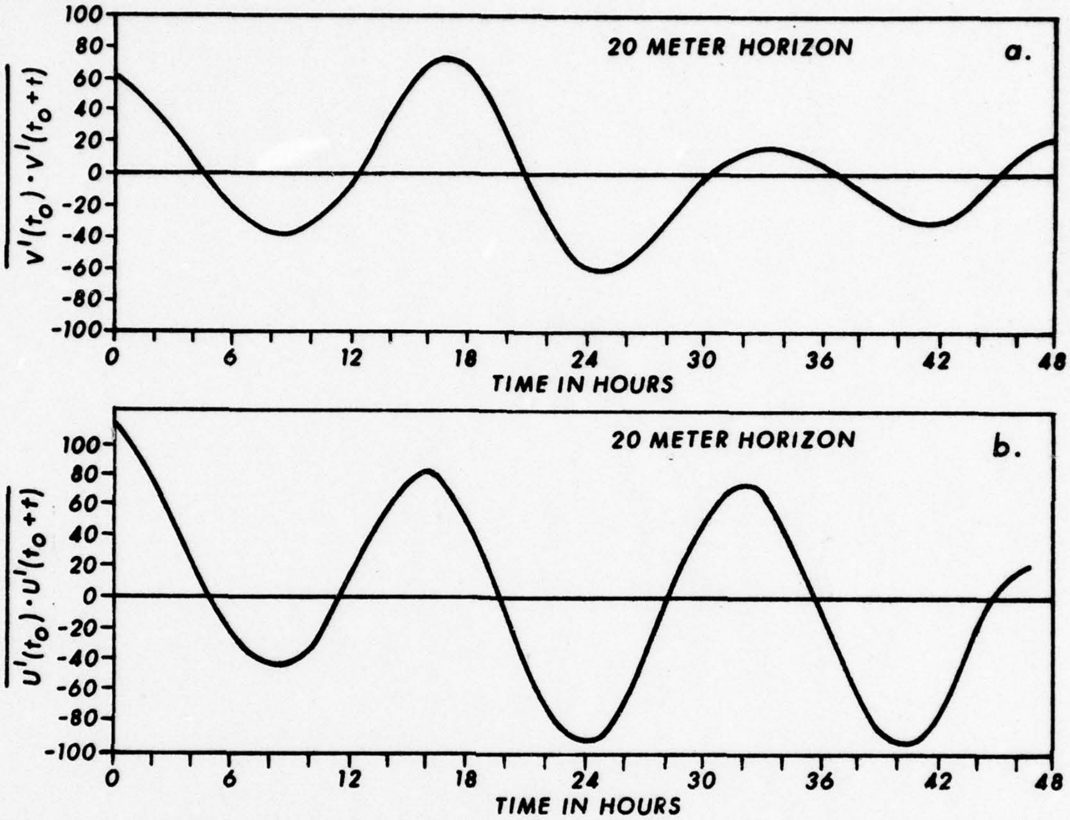

Fig. 1 - Graphs of autocorrelations functions $R_v(t)$ and $R_u(t)$ for the 20 meter horizon and for the selection of initial times as marks numbered 389 - 488

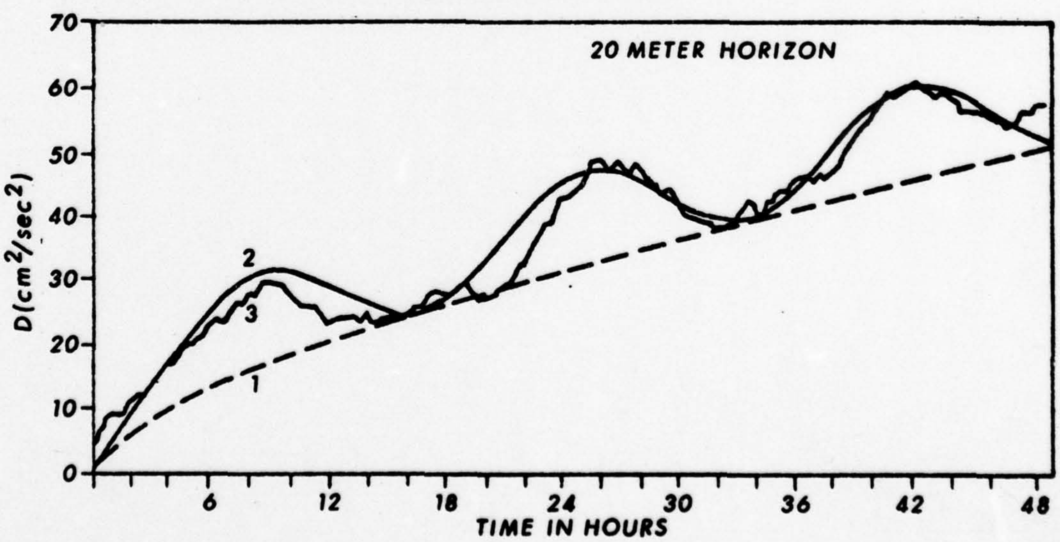

Fig. 2 - The natrually occurring structure function $D_v(t)$ for the 100 meter horizon

1. Theoretical curve of the "2/3 law."
2. Graph of theoretical function (16).
3. Experimental values.

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APPLICATION OF THE THEORY OF RANDOM FUNCTIONS TO THE STUDY OF OCEAN CURRENTS*

by

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In the past few years, among specialists studying ocean currents, and among navigators using the data of these currents, much attention has been devoted to the variability of ocean currents as one of their essential properties. The majority of investigators have shown [3, 6, 9**] that vector current speeds appear as functions of time and that the concept of constant currents, existing up to now in the specialized literature, is highly conditional. One of the authors [6] made even such a conclusion as: "The variability of speeds and direction of currents is the most essential of their characteristics in any part of the sea and at any time of the year. It may be very definitely said that no currents are absolutely constant for any interval of time, and, in some regions of the ocean, the variability of currents has so much significance that currents of any direction are equally probable."

To a certain degree, the variability of currents is undoubtedly conditioned by the influence of tide-generating forces, i.e., there is a tidal nature to the structure of current vectors. However, if one considers currents in non-tidal seas, or if one excludes tidal components, those currents without such components similarly preserve a characteristic temporal variability. Physically, this is easily explained. Actually, currents are agitated by many factors acting in common. All these factors are quite variable with time, such that it is said to be impossible to use their essential features as the principal means to determine instantaneous current vectors. Moreover, one may consider that the speed and direction of currents appear as random functions of time in general. Qualitatively, workers hypothesize that

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**Translator's Note: Original gave the numeral 9 but there were only 8 sources listed in bibliography.

it is possible to accept, with a sufficient degree of precision, that currents which are examined for such intervals of time that seasonal changes of general hydrometeorological conditions are insignificant appear as stationary random functions of time, for the physical conditions of stationarity [5] are fulfilled in that case.

On the basis of observations, it becomes clear that it is permissible to apply the concepts and apparatus of the theory of random functions to the study of ocean currents; more exactly, its most developed section, correlation theory, can be used.

As is well known, random functions are called stationary in the broad sense of the word (in the sense of A. Y. Hinchina), if their mathematical mean and variance are constants, and if their correlation function depends only on the difference of their arguments. For real random functions, the conditions of Hinchina are not exactly realized as a rule, and some approximations are made.

As is well known, the basic characteristics of stationary random functions of time $\xi(t)$ appear in its mathematical expectation $M\xi(t)$ and its correlation function $B(\tau)$ (τ is the lag time). These characteristics are defined by the formulae:

$$m = M\xi(t) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \xi(t)\,dt, \tag{1}$$

$$B(\tau) = M\xi(t)\,\xi(t+\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T \xi(t+\tau)\,\xi(t)\,dt, \tag{2}$$

or, approximately,

$$m \cong \frac{1}{N}\sum_{k=1}^{N} \xi^{(1)}(k\Delta), \tag{1'}$$

$$B(\tau) \simeq \frac{1}{N} \sum_{k=1}^{N} \xi^{(1)}(k\Delta + \tau)\, \xi^{(1)}(k\Delta). \qquad (2')$$

Here $\Delta = \frac{T}{N}$; $\xi^{(1)}$ is a sample of the process; T is the sample length.

Knowledge of these characteristics allows one to decide a series of investigative and practical tasks. In the present article, two of these will be examined: (1) prognosis of currents; (2) apportionment of the tidal composition of total currents.

For these tasks we used material from multi-day buoy stations taken in western parts of the Atlantic Ocean in the process of expeditionary work in 1960. Computations of six-day graphic projections of vector current speeds at a meridian and parallel were made in order that all reduced data be contained in the vector of current speeds, which would not be obvious in further specialization. Computations were produced on an electronic correlator with magnetic recording constructed by A. N. Shakova, and as a result 20 correlation functions were obtained, the normalized average values of which are shown in the diagram*. These functions correspond to currents measured over five days at separations of 100 - 300 miles and at levels of 25, 50, and 100 meters. All curves acquired were good approximations of the expression:

$$B(\tau) = Ce^{-\alpha|\tau|}\left(\cos\beta\tau + \frac{\alpha}{\beta}\sin\beta|\tau|\right). \qquad (3)$$

Here C is the variance ; α and β are constants. (Actual values of α and β lie in intervals from 0.15 to 0.60 per hour and from 0.33 to 0.70 per hour, respectively. The data obtained were sufficient to confirm that the correlation function of the currents in similar regions, at similar times of the year, and at similar depths always has the form (3).)

*Diagram appears at the end of the text.

PROGNOSTICATION OF CURRENTS

From what has been stated it is clear that none of the texts, like atlases, charts, etc., are able to satisfactorily answer a question which has important significance for navigators: What are currents at a certain point at a particular moment? Texts give only the average value of currents--their mathematical mean from which deviations of actual values may be quite great. It is perfectly clear that the question may be fully answered only by the instantaneous measurement of currents and, in the spaces between measurements, there arises the task of prognosticating currents.

Prognosis of currents may be realized by the basic method of linear extrapolation of random functions. This method, worked out by A. N. Kolmogorov, N. Vinerom, and others, is highly reliable and successful from the point of view of utilization of the appendix to the work of Yagloma [4], from which we took all the useful mathematical apparatus used below.

As is well known [4], the correlation function (3) is related to the spectral density:

$$f(\lambda) = \frac{D}{\lambda^4 + 2a\lambda^2 + b^4}, \tag{4}$$

where

$$a = \alpha^2 - \beta^2;\ b = \sqrt{\alpha^2 + \beta^2};\ D = 2\frac{C\alpha}{\pi}(\alpha^2 + \beta^2).$$

By (4) we find the so-called spectral characteristic extrapolation $\Phi_\tau(\lambda)$; the following conditions ought to be satisfied:

a. $\Phi_\tau(\lambda)$ analytic in lower half-plane and grows more slowly in this half-plane than some power of $|\lambda|$ as $|\lambda| \to \infty$;

b. $\psi_\tau(\lambda) = [e^{i\lambda\tau} - \Phi_\tau(\lambda)] f(\lambda)$
analytic in the upper half-plane and, as $|\lambda| \to \infty$,
decays in this half-plane more quickly than $|\lambda|^{-1-\varepsilon},\ \varepsilon > 0$;

c. $\int\limits_{-\infty}^{\infty} |\Phi_\tau(\lambda)|^2 f(\lambda)\, d\lambda < \infty.$

It is not difficult to show that for these conditions it follows that:

$$\Phi_\tau(\lambda) = A\lambda + B, \tag{5}$$

where constants A and B are found from the system:

$$e^{i\lambda\tau} - \Phi_\tau(\lambda) = 0, \tag{6}$$

for $\lambda_1 = \beta + i\alpha$ and $\lambda_2 = -(\beta - i\alpha)$.

After calculations and transformations, we obtain:

$$\Phi_\tau(\lambda) = i\frac{\lambda}{\beta} e^{-\alpha\tau} \sin\beta\tau + e^{-\alpha\tau}\left(\cos\beta\tau + \frac{\alpha}{\beta}\sin\beta\tau\right). \tag{7}$$

Values for the random functions related to $\Phi_\tau(\lambda)$ are extrapolated by the expression:

$$\xi^{(1)}(t+\tau) = \int\limits_{-\infty}^{\infty} e^{it\lambda}\Phi_\tau(\lambda)\, dz(\lambda). \tag{8}$$

Substituting (7) into (8) and having, as is known in the theory of random functions, the equalities:

$$\int_{-\infty}^{\infty} e^{i\lambda t} dz(\lambda) = \xi(t), \tag{9}$$

and

$$\int_{-\infty}^{\infty} e^{i\lambda t} i\lambda dz(\lambda) = \xi'(t), \tag{10}$$

where $\xi(t)$ is the value of the random function at the instant t, and $\xi'(t)$ is its derivative at that same moment, we finally find:

$$\tilde{\xi}^{(1)}(t+\tau) = e^{-\alpha\tau}\left\{\frac{\sin\beta\tau}{\beta}\xi'^{(1)}(t) + \left(\cos\beta\tau + \frac{\alpha}{\beta}\sin\beta\tau\right)\xi^{(1)}(t)\right\}, \tag{11}$$

or

$$\tilde{\xi}^{(1)}(t+\tau) = e^{-\alpha\tau}\frac{\sin\beta\tau}{\beta}\xi'^{(1)}(t) + \tilde{B}(\tau)\xi(t). \tag{12}$$

If $M\xi(t)$ = 0, then, before starting to prognosticate, $M\xi(t)$ is normalized to have a zero mean.

By formula (12) we produced the prognosis of currents at 3 points in the Atlantic Ocean at 1, 2, 3, and 4 hours ahead; for each sample there were calculated five prognosticated values. For all there were acquired 240 extrapolated values, each of which corresponded to an actual value known from measurements.

For example: the predicted currents for one of these stations are shown in Table 1.

The following differences were formed judging the effectiveness of prognosis:

$$\begin{aligned} \Delta_1 &= \tilde{\xi}(t+\tau) - \xi(t+\tau), \\ \Delta_2 &= \xi(t) - \xi(t+\tau), \end{aligned} \qquad (15)$$

where $\tilde{\xi}(t+\tau)$ is the prognosticated value of the current; $\xi(t+\tau)$ is the actual value of the current; $\xi(t)$ is the actual current value at the initial time.

Later Δ_1 and Δ_2 were expressed in terms of σ, the mean square deviation of the data, when we assume the instantaneous value of current to be equal to its average value, and by the formula:

$$\sigma_p = \sqrt{\frac{\sum_1^n \Delta_1^2}{n-1}}\ ; \quad \sigma_{meas} = \sqrt{\frac{\sum_1^n \Delta_2^2}{n-1}} \qquad (16)$$

There were computed the error in prognosis and the error for the case of the current at moment $t+\tau$ having the value it had had at the beginning moment t.

Results of these computations are shown in Table 2.

Moreover, if at the moment $t+\tau$ there is at our disposal the possibility of selection from three values of current: the averaged $M\xi(t)$, the measured $\xi(t)$, and the prognosticated $\xi(t+\tau)$, the most accurate of these is considered to be the prognosticated value (remembering that we always understand the value of the vector projection of current speed on a meridian or parallel by the words "current value".)

Of course, so far, this method is to be applied to currents whose tidal constituents are sufficiently small or have been excluded from the total current. Our further dissertation will be dedicated to finding ways which may be used to exclude the tidal constituents.

ISOLATION OF TIDAL COMPONENTS IN TOTAL CURRENTS

In general, we represent currents in the form of the sum of three functions:

$$\xi(t) = M\xi(t) + \eta(t) + \varphi(t). \tag{17}$$

where $\eta(t)$ is a "pure" random function; $\varphi(t)$ is a nonrandom periodic function of time (e.g., a tidal component).

Not diminishing the generality of our development, we may allow:

$$M\xi(t) = 0,$$

then

$$\xi(t) = \eta(t) + \varphi(t); \tag{18}$$

the function $\varphi(t)$ may be placed in a Fourier series expansion:

$$\varphi(t) = \sum_{k=1}^{\infty} A_k \sin(k\omega t + \theta_k), \tag{19}$$

where

$$\omega = \frac{2\pi}{T};\ A_k = \sqrt{a_k^2 + b_k^2};\ \operatorname{tg} \theta_k = \frac{a_k}{b_k};$$

a_k and b_k are the Euler coefficients of the Fourier expansion of the function $\varphi(t)$. . In this expansion, b_0 would represent $M\xi(t)$

It is easy to show that as a result of the treatment of (17) on a correlator, or, similarly, inserting (17) into formula (2) and (2') we acquire:

$$B_\xi(\tau) = B_\eta(\tau) + \sum_{k=1}^{\infty} \frac{A_k^2}{2} \cos k\omega\tau. \tag{20}$$

The correlation function $B_\eta(\tau)$ of the random function $\eta(t)$ tends to zero as $\tau \to \infty$ (in practice, when τ = 15 to 20 hours, $B_\eta(\tau) \approx 0$) ; members of же $\sum_{k=1}^{\infty} \frac{A_k^2}{2} \cos k\omega\tau$ are undamped, and, consequently, $B_\xi(\tau)$ is likewise not damped as $\tau \to \infty$.

Moreover, the presence of an undamped periodic component in the graph, achieved as a result of treatment on a correlator of sample current, appears as a necessary and sufficient indication of the existence of a tidal current at the investigated point.

On the contrary, the absence of a periodic undamped component in the composition of $B_{\xi}(\tau)$. appears as certification that the tidal current is absent at the data point and that the variability in the current is formed only by random causes.

In the simplest case, when a hodograph of vector speed of tidal current is presented as an ellipse describing the vector current with constant angular speed ω (which may apparently be considered real in the open ocean at great distances from the coast and at great depths), it is not difficult to show that the undamped component in equation (20) has the form:

$$\frac{A^2}{2}\cos\omega\tau. \qquad (21)$$

From the graph of (21), it is easy to select amplitude A and frequency ω so as to construct the function:

$$\varphi(t') = A \sin \omega t',\ t' = t - \frac{\theta}{\omega}, \qquad (22)$$

defining the tidal current.

Clearly, the amplitude of the total current, $\xi(t)$, is always larger than the amplitude of the tidal current $\varphi(t)$.
From (21) and (22), in order to acquire the value of the amplitudes of the tidal current, it is necessary to double the magnitude of amplitude of the undamped component removed from the graph of $B_{\xi}(\tau)$ and then extract the square root. The result is the exact value of the amplitude of the tidal current in the case of an elliptic hodograph and an approximation which becomes more exact the closer the graph of current is to a sinusoidal curve in all other cases.

In treatments of our realizations, the amplitude of the projection of the speed of the total current was 40 - 50 cm/sec, when the amplitude of the tidal component was not in excess of 6 - 8 cm/sec. The graph of the undamped part of $B_\xi(\tau)$ presented a curve which was quite near to a cosinusoid; that certified the nearness of the graph to a sinusoidal curve, which in its turn certified the nearness of the hodograph's form to an ellipse; at last, the acquired value of the amplitude was shown to high accuracy.

If we examine the realization as a tidal current by the ordinary methods of harmonic analysis (for example, by the method of Darwin), the presence of a tidal current with an amplitude up to 40 - 50 cm/sec would be determined. Curiously, the same magnitude for the tidal current would be substantiated by theoretical computations based on the determined hydrodynamic levels in the ocean. For an example, we go to an article by **Boris** [1]. In that article there is a chart of the North Atlantic on which are shown the computed theoretical amplitudes of tidal current speeds. For the region in which the observations employed in this work were produced, theoretical computations gave the magnitude of the amplitude as being on the same order as that which we used. The author of the article [1] assumed that the result of computations of the amplitude does not coincide with calculations from observations because of a defect or insufficiency in the theoretical conditions. In view of the former discussion, it may be considered that the demonstrated discrepancy appears as a consequence not of inadequate theory, but of inadequacies in the method of processing the experimental data, by which the total and not the tidal current was subjected to harmonic analysis. We noticed that errors in the amplitudes of the projection of the tidal current speed inevitably yielded errors in the orientation of the current ellipse.

Moreover, harmonic analysis ought to be preceded by the processing of the realization on a correlator for the purpose of estimating the tidal component. From the preceding, the expansion in a Fourier series of the undamped component ("tail") of the correlation function (20) at once obtains the value of the frequency ω and the amplitude A_k of the terms in the Fourier series. It is more difficult to find the magnitude of Θ_k, which determines the position of the function $\varphi(t)$ at the initial moment. That step may be determined either by the method of successive approximations, or by the formula of Wyrtki [8].

The method of successive approximations in the case of an elliptical hodograph consists of the graph of function (22) combined with the outcome of realization (18) in order to determine, by sight, the most probable position of curve (22) on the time axis. After the generation of the difference function:

$$\eta_1(t) = \xi(t) - A\sin\omega t', \tag{23}$$

which is again generated on the correlator, and, if the combination were successfully accomplished, there should result the acquired correlation function without an undamped periodic component. In cases of adverse combinations, this computation is repeated for some displacement of curve (22) along the time axis; after that, the computation of a different curve $\eta_2(t)$, is produced repeatedly on the correlator until the correlation function is completely deprived of its undamped member. This final version of the function makes it possible to find the tidal component from the simple equality:

$$\varphi(t) = \xi(t) - \eta_n(t), \tag{24}$$

which is then subjected to harmonic analysis.

If there is a Fourier series representation (e.g., for a very complex hodograph), the method of successive approximations adapts itself to a full analog, the only difference being that each time a first harmonic $A_1^n \cos\omega t'$ of the Fourier series is deduced from the decomposed, undamped periodic components of the correlation functions, which are acquired after n processings on the correlator of $n-1$ difference functions.

Wyrtki's formula permits the immediate determination of the angle Θ_k, which locates the position of the corresponding Fourier series (or curve (22)) on the time axis. Therefore, in general, the number of processings of the curves on the correlator is decreased, but the number of curve tracings is necessarily increased for the following:

$$\xi_1 = \xi(t)\cos k\omega t,$$
$$\xi_2 = \xi(t)\sin k\omega t, \tag{25}$$

for each harmonic. The angle Θ_k is acquired by the formula:

$$\operatorname{tg}\theta_k = \frac{M\,[\xi(t)\cos k\omega t]}{M\,[\xi(t)\sin k\omega t]}, \tag{26}$$

where the corresponding mathematical means for the processed curves are determined on the correlator.

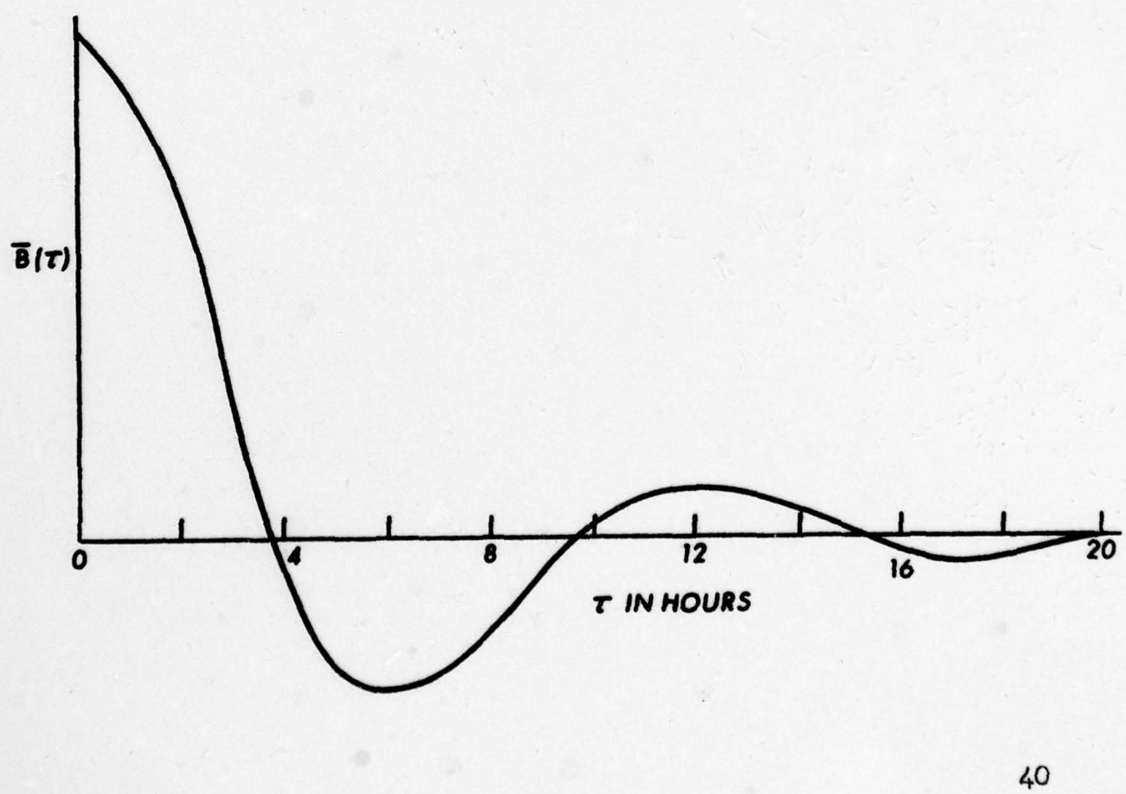

Table 1

PROGNOSTICATED AND MEASURED VALUES OF VECTOR CURRENT PROJECTIONS OF CURRENT WITH CHANGING COMPOSITION

Station No. / Horizon, Meters	Date / Time	Projection	Time of Prognosis Hours	Prognostication $\xi^{(1)}(t+\tau)$	Measured Value $\xi^{(1)}(t+\tau)$	Δ_i	σ_p
1/25	15.VII—1960 / 16.00	Meridian (X)	1	+3,6	+4,0	+0,4	8
			2	+0,7	+1,5	+0,8	12,8
			3	−2,9	−4,2	−1,3	15,7
			4	−6,2	−10,5	−4,3	16,0
		Parallel (Y)	1	+40,5	+41,2	+0,7	3,5
			2	+40,3	+43,0	+2,7	7,0
			3	+38,3	+42,0	+3,7	11,3
			4	+33,8	+38,5	+4,7	20,3
	16.VII—1960 / 00.00	X	1	−11,9	−15,0	−3,1	8
			2	−7,9	−11,0	−3,1	12,8
			3	−5,3	−6,5	−1,2	15,7
			4	−2,6	0,0	+2,6	16,0
		Y	1	+13,0	+18,0	+5,0	3,5
			2	+11,7	+27,0	+15,3	7,0
			3	+10,7	+37,0	+26,3	11,3
			4	+19,7	+43,0	+32,3	20,3
	16.VII—1960 / 08.00	X	1	−20,9	−31,0	−10,1	8
			2	−14,0	−35,5	−21,5	12,8
			3	−7,8	−39,0	−31,2	15,7
			4	−1,9	−40,0	−38,1	16,0
		Y	1	+23,0	+20,3	−2,7	3,5
			2	+20,1	+14,3	−5,6	7,0
			3	+18,3	+7,0	−11,3	11,3
			4	+16,9	+0,3	−16,6	20,3
	18.VII—1960 / 04.00	X	1	−9,2	−11,2	−2,0	8
			2	−6,1	−8,3	−2,2	12,8
			3	−4,6	−8,3	−3,7	15,7
			4	−2,7	−11,5	−8,8	16,0
		Y	1	+10,2	+11,3	+1,1	3,5
			2	+10,1	+13,3	+2,2	7,0
			3	+8,8	+13,5	+4,7	11,3
			4	+7,9	+11,0	+3,1	3
	19.VII—1960 / 16.00	X	1	−10,3	−16,0	−5,7	20,8
			2	−6,3	−13,0	−6,7	12,8
			3	−4,4	−10,0	−5,6	15,7
			4	−2,1	−4,0	−1,9	16,0
		Y	1	−19,4	−17,5	+1,9	3,5
			2	−14,4	−14,0	+0,4	7,0
			3	−15,8	−6,5	+9,3	11,3
			4	−14,4	−1,0	+13,4	20,3

Table 2

σ	$\tau=1$ hour		$\tau=2$ hours		$\tau=3$ hours		$\tau=4$ hours	
	σ_p	σ_{meas}	σ_p	σ_{meas}	σ_p	σ_{meas}	σ_p	σ_{meas}
100%	19%	31%	45%	58%	74%	83%	80%	100%

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SOME RESULTS OF INVESTIGATIONS OF TIDAL CURRENTS IN THE OPEN PARTS OF THE PACIFIC OCEAN *

by

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For the explanation of the character and the strength of tidal currents in the open parts of the Pacific Ocean, the I/S U SHOKALSKII, on her second cruise (November 1960 - January 1961), at a line of station points in the northern central regions, produced moderately broad studies; five days of cyclical current measurements were analyzed by the scheme proposed by A. I. Dubaninin[1]. Investigators employed electromagnetic current meters (EMIT). In the course of a day, the ship cruised by a plan of two circuits every three hours. Daily cycles were accomplished for nearly four days.

Harmonic constants, found by the conjugate method, gave satisfactory agreement between calculated and experimental data in 70 percent of the cases.

It was found that there was better agreement for waves of semi-diurnal period than diurnal.

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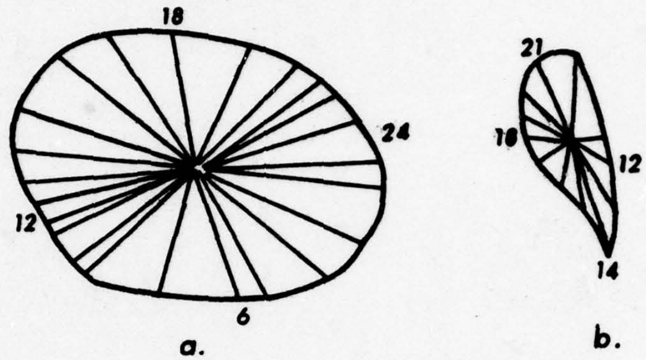
*The article appears in Oceanology, Vol 2, No. 2, 1962, pp. 260-261.
[1]Dubaninin, A. I., 1955, "Certain Characteristics of Tides by Short Duration Observations," Tr. GOV. Oceanog. Inst., Issue 30 (42).



Tidal Current Ellipse in Open Oceans by Observations

a - Diurnal Wave; b - Semi-Diurnal Wave

Magnitudes of current speed were acquired with the EMIT; the absolute correction "K" was not introduced.

Results showed that tidal currents obtained significant speeds (up to 60 - 70 cm/sec) on the surface.

The character of tidal currents is irregular, diurnal. The ellipse of daily currents is nearly circular, as expressed by the vector diagram.

Besides evidence from this five-day period, obtained under difficult celestial navigation conditions, data were taken for one-day periods in different regions: in the zone of the Northern and Southern Tradewind Currents and in the zone of the Equatorial Countercurrent. Results of these investigations, as in the five days listed formerly, confirmed the existence of different types of tidal currents in the indicated regions of the Pacific. Observers also confirmed the existence of constant currents present in the regions cited, and only the Equatorial Countercurrent showed few deviations to the north and in greatest speed as compared with average data for the season. Simultaneous observations of hydrological and hydrodynamical elements showed significant oscillations.

CONCLUSIONS

1. Tidal currents in the open ocean are of enough significance to cause the measurement of steady currents to be approached carefully.

2. With EMIT, it is possible to measure tidal currents in open and deep waters for the purpose of using their harmonic constants to distinguish residual (constant) currents.

3. In regions where significant tidal currents exist, there occur similar, significant oscillations of hydrologic and hydrodynamic elements; it is necessary to take cognizance of them in different types of computations.

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ON THE PERIODS OF INTERNAL WAVES IN DEEP INLAND SEAS*

by

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In the past decade there has grown a collection of works about one of the least studied oceanological problems, i.e., the investigation of the character, distribution, and origin of so-called "internal waves".

The hydrosphere presents itself as a medium consisting of a series of water layers with differing density. In such a layered structure of water, when the interfaces between layers are under the action of various excitation forces (tidal forces, atmospheric disturbances, etc.,) internal waves are generated with amplitudes attaining decameters and sometimes hundreds of meters. Naturally, such "appreciable" vertical changes in the water mass play a large role in the dynamics of the world ocean. The significance of internal waves as factors contributing to water variations and the transmission of various hydrological characteristics to deep layers is very great, not only for the hydrological regime of basins but for their biological domains as well by aiding the distribution of various things having importance to living organisms. Knowledge of the magnitude of internal wave parameters and of the laws of their variability also has value for submarine navigation, for hydroacoustic prognosis, for the needs of the fishing industry, and for other purposes.

In various parts of the world ocean there exist internal waves with different characteristic parameters depending upon their dimensions, the configuration of the coasts, and, especially, temperature and salinity fields, which create the density stratifications that are typical for each water basin.

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*Published in Oceanology, Volume III, No. 6, 1963 (UDK 551.465, pp. 962-9).

The present work contains the analysis of germane data, acquired at two multi-day stations in one of the deep, inland seas at distances of 6 and 28 miles from the shore for the purpose of determining the periods of internal waves. These stations were occupied in the spring for periods of 4 - 5 days and were synchronozed between two ships which were deployed along a track normal to the shore. There was a two-hour interval between observations.

For the analysis of our data acquired at these stations, we studied time-varying vertical oscillations of density. We took the average values of density at standard horizons for the period of observation. Moreover, time-varying oscillations of these isopycnals were also related to standard horizons. Similarly, the representation of the studied quantities makes it possible to compare the parameters of internal waves at one or the other of these horizons in various regions of the sea in various seasons.

Deep inland basins possess special conditions for the generation of internal waves. In observed internal waves in the sea, tidal forces do not appear to be the principal factor causing oscillations of water particles. The greatest amplitude of tides here is on the order of 10 cm (at the surface); the character of the tides is irregular, semidiurnal. Of course, the existence at considerable depths of significant vertical oscillations attaining 100 -150 meters, as seen in the analysis of the data of the expedition, reflects the non-tidal origin of these fluctuations. V. V. Schulaykin and A. D. Dubrovelcki were the first to show the seiche origin of internal waves in inland seas.

It is well known that the excitation of internal waves in any place of the sea can be caused by small impulses due to changes in the atmospheric pressure, changes in the wind field, seismic shocks, etc. Waves caused by various impulses are found in various stages of development or attenuation, are dispersed in all possible directions, and are superimposed upon the unattenuated periodic oscillations of tidal character; thus, systems of internal waves are generated in a given basin. It was quite obvious that, even for a repeat of one of these synoptic situations, the quantitative state of the spectra of internal waves may change for the investigated place. At the same time, it seems that there is an inherently determined combination of periods for each section of the sea.

Due to the fact that internal waves by their nature and mechanisms develop analogously to surface waves, we recognized the possibility of

using one of the methods for quantative estimate of characteristics of sea-surface waves for the determination of internal wave periods, namely, the method of the theory of random functions. The basic characteristics of random functions appear as the average value (mathematical mean) for the correlation function. It is to be noted that for simple computations, for known admissibility, we used properties of what are called "stationary" random functions. In this case, the wave function is given a zero mean value, and, for the characterization of the random function, it is sufficient to compute its correlation (autocorrelation) function.

The correlation function $R(\tau)$ of random function $f(t)$ appears as a quantitative measure of the persistence of the random function, when samples have been separated one from the other by the magnitude τ ; it is approximately determined by the expression:

$$R(\tau) = \frac{1}{2T} \int_{-T}^{T} f(t) f(t+\tau) dt. \tag{1}$$

The value of the correlation function for $\tau = 0$ is the variance of the random process.

In practice, a small number of values for the analysis of surface as well as internal waves are available to perform the calculation of their correlation function and are related to the Fourier transform (in agreement with the well-known theory of Hinchina) for the so-called spectral density $S(\omega)$. Spectral density and correlation functions are related to one another by their Fourier transforms as follows:

$$S(\omega) = \frac{2}{\pi} \int_{0}^{\infty} R(\tau) \cos \omega \tau d\tau; \tag{2}$$

$$R(\tau) = \int_{0}^{\infty} S(\omega) \cos \omega \tau d\omega; \tag{3}$$

where ω is the angular frequency. The function $S(\omega)$, or spectral density, appears as the frequency characteristic of the random function $f(t)$, determining for the process its distribution of energy according to spectral frequency.

In our case the random function $f(t)$ is mixed with the undamped periodic $f_1(t)$. The correlation function of the sum $f(t) + f_1(t)$ will also be the sum $R(\tau)+R_1(\tau)$, in which $R(\tau)$ would decay to zero with an increase in τ, and $R_1(\tau)$ will be periodic with the frequency of the germane function [2] and [4]. In the case of mixed functions formula (2) is also applied.

A mechanical correlator was used for the acquisition of the correlation functions, since numerical determination is tied to very unwieldy computations. By polar graphical representation of our time functions by two indices with changing τ in friction mechanism of the correlator we created the continuous summary product $f(t)\cdot f(t+\tau)$, and in our counters, after the circular curve, we acquired a magnitude proportioned to the integral $\int_0^{T-\tau} f(t)\cdot f(t+\tau)\,d\tau$, where T is the duration of the function in hours. The approximate value of the correlation function is acquired, finally, if we multiply the value of the coefficient on the device by the value $\frac{1}{T-\tau}$. The integration steps were taken as 1 hour 40 minutes.

Computation of $S(\omega)$ was produced for the ordinate of density temporal oscillations relative to the average horizon at 25, 50, 75, 100, and 150 meters for the offshore station and at 25, 100, 150, 200, 300, and 422 meters for the inshore station. For this computation, the following approximate formula was used:

$$S(\omega) \approx \frac{2}{\pi}\int_0^T R(\tau)\cos\omega\tau\,d\tau. \qquad (4)$$

The integration was produced numerically by the formula:

$$S(\omega) \approx \frac{2\Delta t}{\pi}\left(\frac{R_0}{2} + \sum_{1}^{m-1} R_n \cos\omega n\Delta t + \frac{R_m}{2}\cos\omega m\Delta t\right); \qquad (5)$$

where R_0 is the value of the correlation function for $\tau = 0$; R_n is the value of the correlation function over equal intervals of Δt ; and m is the quantity of pieces Δt .

Since $\omega = \frac{2\pi}{T}$, for the convenience of computations of $S(\omega)$ we changed the value of T . Up to T = 22 hours, ΔT was taken to be 5 minutes; for $T >$ 22 hours, ΔT was taken as 1 hour 40 minutes. For the precise location of the maximum spectral density, the intervals ΔT were increased two and even four times.

From the relations (2) and (4) it is quite obvious that the exact computation of the spectral density depends basically on the exact determination of the correlation function. The latter clearly depends on the maximum value of τ , the length of integration T, the magnitude of the integration step, and on the instrumentation errors of the apparatus. As far as work on the mechanical correlator is concerned, the two operations provide an opportunity for very large errors to arise from calculations because of the subjective character of compression.

Because of the approximately determined correlation functions, errors in observation, etc., $S(\omega)$ may have a negative value, which it should not, for some value of $\omega = \frac{2\pi}{T}$. The spectrogram in Fig. 1 may serve as an example. The value of $S(\omega)$ is negative at T, equals 5.8, 7.5, 16.6, and 42.0 hours. Clearly, for the absence of the enumerated errors at some of these frequencies, the ordinate $S(\omega)$ ought to be either equal to zero or nearly equal to zero. However, in spite of these deficiencies, which occur in computating the magnitude of the spectral density with the aid of the method summarized, it is possible to acquire, from the spectra, the periods of the complex oscillations which compose the waves and which we observe in nature.

The accompanying tables present all the significant periods we acquired in the interval of 6 hours to 3 days at the offshore and inshore stations. (Tables 1 and 2).

Internal wave periods are quite well grouped according to the vertical. Ten isolated groups of periods are distinguished by characteristic values. The short periods - 5.8, 6.7, 7.5, and 8.5 hours - belong to Group I. They were present at both stations from 25 to 422 meters deep. These periods compare completely with the well-known periods for seiches of the sea surface in this region. Group II contains the components with periods of 9 - 10 hours, which are also of seiche origin.

Group III is of great interest because its periods conform to periods of semidiurnal tidal character, as observed in the interior of the seas. In spite of the insignificant (compared to the former) amplitudes of the surface tides, they were later observed to be influential in all layers of water. From the examination of the tables, it is seen that waves with a semidiurnal period exist at all the horizons we investigated.

The existence of waves with periods of Group V is similarly predicted. They are oscillations with the inertial period. In agreement with formula $\left(T = \frac{12}{\sin\varphi}\right)$; the period of internal inertial waves for the average latitude equals 17 to 18 hours.

Further, from the analysis of the tables it is to be noted that the nature of the inshore and offshore stations characterizes their groups of waves. Thus, at the offshore station there exist components with 24-hour periods (Group VII) at all horizons. At the same time at the station located 6 miles off the coast, the components with this period did not exist. However, at this same station at all depths there existed components with periods of 19.5 - 21.5 hours, which were not revealed at the offshore station. It is very possible that these two groups of components were generated by one and the same cause, i.e., from the diurnal cycle of change.

Apparently, the phenomenon of a diurnal wave is transformed and its period becomes equal to 19.5 - 21.5 hours according to its proximity to the coast. In our opinion it seems a probable hypothesis that causes of differences in and the absence of diurnal components at the inshore station and the absence of components with

periods of 19.5 - 21.5 hours at the offshore station are explained by the disposition of the stations according to their relationship to the nodes of standing oscillations with analogous periods.

From the analysis of the preceding group of components with long periods, there is likewise clearly seen a distortion (decreasing) of periods for the approach of waves to the shore.

As far as the series of observations at both stations are concerned, the internal wave periods did not exceed 5 days; the maximum possible periods were resolved as 2- and 3-day components. However, it is necessary to note that the accuracy of these determinations was very low.

All enumerated groups of oscillations in the observed inland basin have the character of seiche oscillations and a meteorological origin as the basic cause. For all more or less significant atmospheric perturbations (expressed in the changing plot of pressure and wind), water layers are subjected to an impulse, and oscillations may arise in them with the period (cycle) of the changing pressure and wind.

In Fig. 2 there are presented: (1) a plot of pressure and wind (the upper curves) at the coastal station at the time the multi-day stations were produced and for the 12 preceding days (Fig. 2a); (2) a plot of pressure at the offshore station (Fig. 2b); and (3) curve of sea level according to hourly tide gauge readings at the given shore station (Fig. 2c).

As clarified by the analysis of surface weather charts and, in some measure, from the presented patterns, in an average 25-day period over the region, a deep cyclone (1006 мб) is found. In the immediately succeeding days there occurred a rare change in the synoptic situation. There was a cycle span of high pressure at this place. It was virtually destroyed in one day. From the 27th to the 2nd, a series of cycles traveled across this region at intervals on the order of 2 days. On the 3rd there was produced an unusual increase of pressure for records collected in the region of the station at the crest of the high pressure, which to the end of the 3rd formed in a nucleus. In the following days, synoptic observations determined the intensity of the nucleus and the influence of the trough of low pressure, located to the northwest of the region. By observing Fig. 2, the presence in the plot of pressure and wind is to be noted; also, the oscillation of sea level of 3-day periodicity is a condition due to the strong influence of the nucleus

of high pressure or the trough of low pressure. The wind in the region of the stations and at the nearby shorepoint was very weak and variable during the observation period.

It is quite obvious that two-day and three-day components of internal waves in the series taken were produced from an exchange of atmospheric impulses. Along with important atmospheric perturbations stimulating long-period oscillations, in all stretches of the pressure curves viewed there were small perturbations with a variety of periods, which nonetheless may have initiated seiche oscillations with periods noted in the tables.

It is very well shown that such agreement may serve as the correlative tie between time plots of atmospheric pressure and density at the 25-meter horizon for the analyzed, synchronized stations. Computation of correlation coefficients were made by the formula:

$$R(\tau)=\frac{\sum_{n=1}^{N-\tau} P_n\sigma_{n+\tau}-\frac{1}{N-\tau}\sum_{n=1}^{N-\tau} P_n \sum_{n=\tau+1}^{N} \sigma_n}{\sqrt{\left[\sum_{n=1}^{N-\tau} P_n^2-\frac{1}{N-\tau}\left(\sum_{n=1}^{N-\tau} P_n\right)^2\right]\left[\sum_{n=\tau+1}^{N} \sigma_n^2-\frac{1}{N-\tau}\left(\sum_{n=\tau+1}^{N} \sigma_n\right)^2\right]}}, \qquad (6)$$

where N is the number of data points, τ is the delay time, P is the pressure, and σ is the density condition. It seems that for Station II* (inshore) the largest correlation coefficient equaled 0.95 for a lag time of nearly 29 hours. At the same time, Station II (offshore) had a maximum correlation coefficient equal to 0.84 without delay time.

*Translator's Note: This is probably a typographical error and should be Station I.

The plots of pressure at both stations are practically one and the same, and, therefore, for the attempt of constructing the correlative tie between density oscillations at the 25-meter horizon at both stations, it follows to anticipate the largest correlation coefficient for a lag time on the order of 29 hours. Computations of such correlative ties show that the greatest correlation coefficients were equal to 0.71 for a delay time on the order of 34 hours. Moreover, density oscillations at these depths in the region of the inshore station were delayed, relative to similar oscillations in the region of the seaward stations, by 34 hours. It was possible to measure the speed of propagation of the observed waves by knowing their delay times. Assuming that the wavefront propagates normal to the coast, its speed would equal 36 cm/sec; if the front propagates at an angle less than 90°, then the speed of the internal wave would be less than 36 cm/sec.

For the isolation of periods of the basic components of the waves, besides the formerly stated method, we used the Fourier method of auto-correlation. That method isolates the components in order of their significance, i.e., by the value of their amplitudes from a series of observations. Formerly [1] we used the contents of this briefly stated method.

Periods obtained by one or the other method were found to be in sufficiently good agreement. We will examine the following example. On the 100-meter horizon at the offshore station the Fourier analysis gave the value of the period of the predominant component as 26.9 hours (amplitude 6.3 meters), and the second most significant component had a period of nearly 80 hours. The analogous period acquired by the second method were equal to 25.8 and 69 hours.

For the 50-meter horizon, the principal period acquired was equal to 25.7 hours (amplitude 2.2 meters) by the first method, 24.2 hours for the second method.

By an analogous comparison with results at 300 and 422 meters of the inshore Station I, it was shown: (1) the Fourier method determined the period of the basic components at the 300-meter horizon — 21.1 hours (16.9 meter amplitude), and at the 422-meter horizon — 20.4 hours (12.8 meter amplitude); (2) comparable periods acquired by the application of the spectral density formula were 20.3 hours (300-meter depth) and 21.1 hours (422-meter depth).

Therefore, it must be noted that there are good comparative results acquired by both methods.

In the majority of cases [1, 5], application of the Fourier method gives good, reliable results. However, when the predominant period (according to amplitude) of the next largest component exceeds the length of the time series, or when the time series is less than one-half to two times the length of the period, computations by the Fourier method appear difficult for the determination of the periods of the remaining components. Such difficulties emerged in our computations of periods for the 100, 150, and 200 meter horizons at Station I (inshore) by these methods. By the use of the computational method for spectral density, these difficulties are not encountered, and we acquired all possible periodic components existing at the time of observations. However, it is noted that the long periods acquired were quite approximate due to the insufficient extent of the stations.

For suitable conditions we considered it expedient to use either of the methods for the acquisition of the frequency distribution of internal wave components in similar regions of the sea.

In addition to spectra of internal wave periods at various depths, periods of changes in atmospheric pressure and sea level were also determined. The data obtained are reduced in Table 3. As a result of the comparisons in the three tables, there is an obviously good relationship between periods of oscillation on all the previously mentioned horizons and periods of sea level and pressure changes. The results of the comparisons once more confirm the opinion that in deep inland seas atmospheric impulses are shown to influence water layers very significantly.

On the basis of all the foregoing one may make the following conclusions:

1. Each of the observed curves of density isopleths (for the offshore as well as the inshore station) appears in the nature of a complex oscillation, consisting of a sum of vertical oscillations of various periods (as in Tables 1 and 2).

2. By data analysis using the Fourier method and the distribution of spectral density, it was shown that, in general, the combined

oscillations appeared to be dominated by components with periods of 21 - 24 hours and periods on the order of 3 days.

3. The principal factor in the phenomena of internal waves in deep inland seas appears to be the variability of atmospheric conditions, e.g., pressure, wind regime, etc. In the cases observed, because of the effect of the particular positions of the stations in their proximity to the center of the barometric phenomena, observed winds were very weak and variable in direction. Accordingly, in the given case, the principal wave excitation factor appeared as the variability of atmospheric pressure.

4. The semidiurnal tidal component exists at all horizons at both stations and, in spite of this, it does not appear to be dominant but has a large significance in the mixing of water at great depths.

5. For the objective estimate of phenomena, it is necessary to limit the isolation to one or two components of the waves. The spectral analysis of internal waves makes it possible to determine the degree of influence of one or another of the forces exciting internal waves with corresponding parameters. Knowing the state of these forces and the laws governing them, it may further be possible to construct laws about the propagation of internal waves. Finally, as noted earlier, this knowledge has great practical value.

The study of the oscillations of internal seiches in deep inland seas is of interest because water movement in the vertical and horizontal directions at great depths is accompanied and caused by them.

For the acquisition of more complete and reliable data for the quantitative expression of the components of internal waves, it is necessary to occupy multi-day stations for a sufficient duration not only inshore but in deep water parts of the sea and with the participation of several ships working in synchronization according to a special program.

In conclusion, the authors express deep gratitude to A. G. Kolesnikov for valuable discussions and help in the work, and also A. F. Sokonnov, who conscientiously executed a valuable part in the laborious computations.

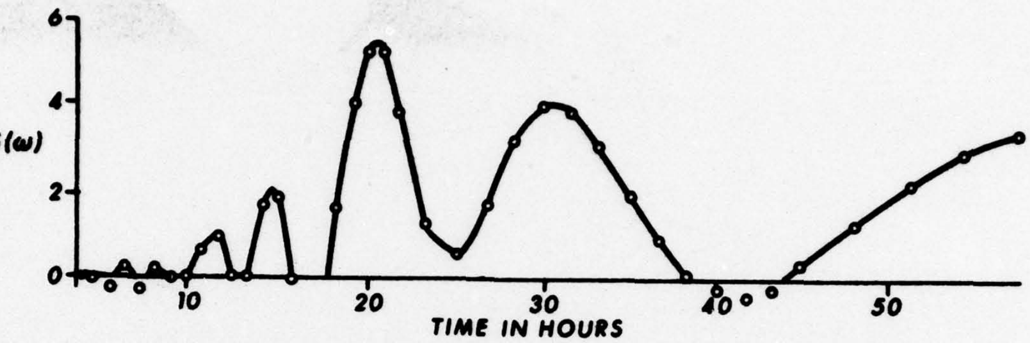

Fig. 1 - Distribution of spectral density on the 300-meter horizon at the station closest to shore

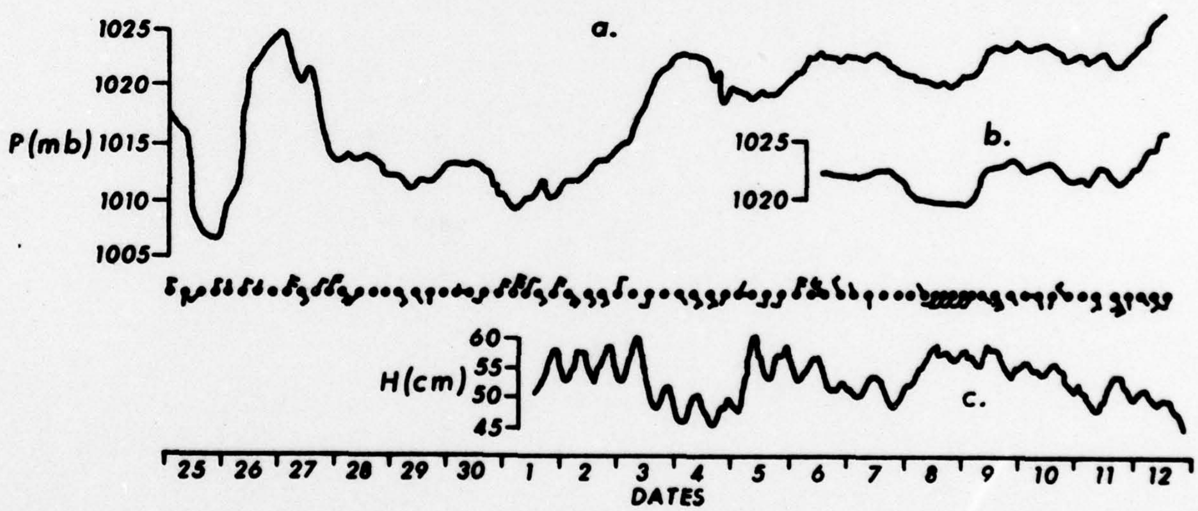

Fig. 2 - Changes with time in meteorological elements and sea level according to data from coastal and open-sea stations

a. Change in pressure and wind at coast station
b. Change in pressure at sea station
c. Change in sea level at shore station

Table 1

PERIODS OF INTERNAL WAVE COMPONENTS
AT INSHORE STATION, IN HOURS

Horizon, m	Groups I		II	III	IV	V	VI	VII	VIII	IX	X
25			9,2	11,5	15,0		19,5			40,8	
100	5,8			10,7	14,5		19,6		30,5		Nearly 3 days
150		7,3	9,2	11,9		16,5			29,3		
200		8,2		11,5	14,5	18,3	21,5		31,7		
300	6,7	8,5		11,5	14,7		20,3		30,7		
422	5,8	8,5		11,7	15,0		21,2				

Table 2

PERIODS OF INTERNAL WAVE COMPONENTS
AT OFFSHORE STATION, IN HOURS

Horizon m	Groups I		II	III	IV	V	VI	VII	VIII	IX	X
25	6,7	8,7		10,8	14,5	18,5		23,7		41,5	Nearly 3 days
50		8,5		11,5	15,0	18,8		24,2	33,3	42,0	
75		7,5		11,6	15,0	18,8		23,7	33,0	45,0	
100			10,0	13,2	15,7		19,6	25,8	37,5		
150			9,0	12,2		17,8		24,2		40,7	

Table 3

PERIODS OF SEA LEVEL AND ATMOSPHERIC
PRESSURE OSCILLATIONS, IN HOURS

Sea Level	5,8 7,8		12.5	14.2	17.2	20.0	24.2	35.0	45.0 55.0	Nearly 3 days
Atmospheric Pressure		10,0		15,0	19,0	20,8 22,5	25,0 27,5	30,6 34,5	46,5 57,0	74,0

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